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AN ANALYSIS OF FREE-RUNNING MODEL TRAJECTORIES OF THE EXPERIMENTAL SUBMARINE SST

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CONTENTS

NOMENCLATURE

Symbol	Dimensionless Form	Definition
	a_t	Effective aspect ratio of tail appendage.
A	$A_t = A/\ell^2$	Area of tail.
I_y, I_{y_o}		Moment of inertia of body and of displaced fluid about the y-axis.
	k_1, k_2, k'	Hydrodynamic mass coefficients.
ℓ	$\ell' = 1$	Characteristic length of submarine.
m_o	$m_o' = m_o/\frac{1}{2}\rho\ell^3$	Mass of submarine.
	$m_2' = m_o/(1+k_2)/\frac{1}{2}\rho\ell^3$	Mass coefficient.
M, M_1, M_2	$M' = M/\frac{1}{2}\rho\ell^3 V^2$	Hydrodynamic moments about the y-axis through the c.g.
M_q	$M_q' = M_q/\frac{1}{2}\rho\ell^4 V$	Derivatives of moment component with respect to angular velocity component q, velocity component w, and stern plane angle δ.
M_w	$M_w' = M_w/\frac{1}{2}\rho\ell^3 V$	
M_δ	$M_\delta' = M_\delta/\frac{1}{2}\rho\ell^3 V^2$	
	$n_y' = (I_y+I_{y_o} k')/\frac{1}{2}\rho\ell^5$	Moment of inertia coefficient about the y-axis.
q	$q' = x_2(s) = q\ell/V$	Angular velocity component relative to y-axis.
t	$s = tV/\ell$	Time
$u, v, w,$		Velocity components of origin of body axes relative to fluid.
V		Velocity of origin of body relative to the fluid.
Z, Z_1, Z_2	$Z' = Z/\frac{1}{2}\rho\ell^2 V^2$	Hydrodynamic normal force, positive downward.

NOMENCLATURE
(continued)

Symbol	Dimensionless Form	Definition
Z_q	$Z'_q = Z_q / \frac{1}{2}\rho \ell^3 V$	Derivatives of force component with respect to angular velocity component q, velocity component w, and stern plane angle δ.
Z_w	$Z'_w = Z_w / \frac{1}{2}\rho \ell^2 V$	
Z_δ	$Z'_\delta = Z_\delta / \frac{1}{2}\rho \ell^2 V^2$	
$Z_{tail} = Z_t$		See Equations 5 and 6, and text.
	z_o	Depth of c.g. from horizontal reference line.
α	$\alpha = x_1(s)$	The angle of attack.
α_{tail}		The local angle of attack at the tail.
$\delta_s = \delta$	δ	Angular displacement of stern planes, positive trailing edge down.
θ	θ	Angle of pitch or inclination of the body axis from the horizontal.
ρ	$\rho' = 1$	Mass density of water.

INTRODUCTION

The theoretical problem of predicting the course which a ship follows in response to a prescribed stern-plane or rudder motion has not been satisfactorily solved. A satisfactory solution of this problem would have considerable practical value, particularly for the motion of a submarine where "maneuverability in depth" is so important. The theory is unsatisfactory in the sense that trajectories describing the motion cannot reliably be predicted from the results of captive model tests. The commonly accepted reason for this failure is that the hydrodynamic characteristics which are determined from captive model tests are not known with sufficient precision or reliability. This reason is quite plausible in view of the discrepant results which are often obtained from different model tests of the same prototype.

The present study was undertaken with the object of ascertaining whether it is possible to combine information from captive model tests with information from free-running model tests for the purpose of constucting hydrodynamic characteristics which may be introduced into differential equations of a given form, such equations being used to characterize the motion, and thus allowing one to predict new trajectories. Only hand computation methods have been used in this investigation.

Although no very fixed conclusions can be drawn, it appears that more accurate free-running tests (as well as captive model tests) are required to obtain positive results. However, it is misleading to imply that nothing of value can be learned from this type of approach. One can, in fact, roughly predict some trajectories and, given enough patience, one could, perhaps, continue to modify the equations so that they fit more and more trajectories. It is known that nonlinear differential equations must be used to characterize the motion and it is, therefore, evident a priori that theoretically there must always exist some ambiguity regarding the validity of the equations. However, from a practical point of view one may say that if equations have been constructed which have as one solution a given trajectory, then these equations should be approximately valid for trajectories which are not too different, i.e., trajectories which do not involve a different type of maneuver and do not involve large differences in the magnitudes of any of the

parameters which directly influence the motion.

GENERAL THEORY

A great deal has been written on the theory of the motion of a solid body through a fluid and here, as elsewhere, the usual assumptions of motion in undisturbed water and in a plane (in this case vertical) are made. The coordinates describing the motion of the center of gravity are taken along, and perpendicular to, the longitudinal axis of the ship. This is customarily done because the hydrodynamic inertia forces, which depend on the geometry of the body, are most simply expressed in this form. We are interested in a description of the motion in a direction perpendicular to the main line of motion as the ship, being self-propelled, does not accelerate or decelerate appreciably along the trajectory while turning. The viscous forces, which affect flow mainly in the boundary layer, are therefore neglected as they have little influence on the motion except insofar as they determine the propeller force required to maintain a constant speed. The coordinate system, terminology, and notation follows that prescribed and set forth in Ref. 1. The general equations of motion have been derived in Refs. 2, 3, and 4, and a complete derivation will not be given here. However, some description of some of the various forms the equations may take is necessary to understand the significance of the empirically determined hydrodynamic forces.

Equations of Motion

The classical equations of motion for a body of revolution with inertia coefficients k_1, k_2 and k', moving in one plane through an ideal fluid are, (see Ref. 2)

$$\begin{aligned} m_o(1+k_1)\,\dot{u} + m_o(1+k_2)\,w\dot{\theta} &= X_1, \\ m_o(1+k_2)\,\dot{w} - m_o(1+k_1)\,u\dot{\theta} &= Z_1, \\ I_y\,(1+k')\,\ddot{\theta} + m_o(k_1-k_2)\,uw &= M_1, \end{aligned} \tag{1}$$

where X_1, Z_1, and M_1 represent external and/or hydrodynamic (or hydrostatic) forces arising in part from the nonideal character of the fluid. There may be included in X_1, Z_1, and M_1 forces arising from the rudder,

stern plane, or the tail fins, and gravity or buoyancy forces.

If the body is self-propelled, and has not too large an angle of attack, then the "u" component of the velocity is considered to be essentially constant, and the first equation contains only second order terms and is eliminated. This assumption results in two equations for w and θ.

It may be noted at this point that the dependent variable w is sometimes replaced by the angle of attack α. When the latter is small and when the speed V is essentially constant, the relations $u \cong V$, $w \cong V\alpha$, allow a simple transformation of the dependent variables.

The mass m_o, and the inertia I_y are quite unambiguous. The inertia coefficients k_1, k_2, and k' may, in theory, be calculated from the geometry of the body, and ordinarily an estimate is obtained in this way. However, the hydrodynamic forces represented by these coefficients are only a part of the total hydrodynamic forces and sometimes the equations are written in a form such that on the left hand side one has only the body inertia forces while on the right hand side one has the total hydrodynamic (and other) forces, e.g.,

$$\begin{aligned} m_o(\dot{w} - u\dot{\theta}) &= Z, \\ I_y \ddot{\theta} &= M \end{aligned} \qquad (2)$$

The hydrodynamic forces and moments, Z and M, will depend not only on w and $\dot{\theta}$ but also on the "accelerations" $\dot{w}$ and $\ddot{\theta}$. Furthermore, as has already been pointed out, there will be included in Z and M the stern plane and fin effects as well as hydrostatic forces, consequently some discussion of these forces is in order.

Forces and Moments

The mathematical formulation of the problem reduces to a detailed specification of what is to be included in, and what excluded from, the terms Z and M.

In the first place, one may point out that besides the five variables, $\dot{w}$, $\ddot{\theta}$, w, $\dot{\theta}$, and u, there is the stern plane angle $\delta_s(t)$, and that, strictly speaking, Z and M depend functionally on all of these quantities in a very complex nonlinear way. Several simplifications of the problem are necessary and the first of these is the assumption that only linear terms in the

accelerations $\dot{w}$ and $\ddot{\theta}$ are involved and that the principal ones, of these terms, have the values predicted, e.g., $Z_{\dot{w}} = -k_2 m_o$, $M_{\ddot{\theta}} = -k'I_y$, whereas the coupling terms $Z_{\ddot{\theta}}$ and $M_{\dot{w}}$ are negligible. A second simplification results from the assumption that all hydrodynamic forces are proportional to u^2, with the consequence that, except for hydrostatic forces or moments, the equations may be made independent of the speed. The present discussion relates to neutrally buoyant dynamically stable bodies with a small metacentric height and traveling at high speed so that hydrostatic forces and moments are neglected relative to hydrodynamic ones.

With these simplifications the functions Z_1 and M_1 are functions only of the three variables w, $\dot{\theta}$, and $\delta_s = \delta$, the first two depending implicitly, the latter explicitly, on time. The subscript s on δ will henceforth be omitted as this should cause no confusion.

It is known that within the range of variables which occurs, the functions Z_1 and M_1 are "nonlinear" in all three variables. However, in view of the explicit dependence of δ on t it is considered that the problem is analytically unapproachable without a linearization with respect to δ. Such a linearization may, of course, be done in several different ways. In the present paper the initial slope is taken as the basis for the linearization. To summarize then, we have

$$\begin{aligned} Z &= Z_1 - m_o k_2 \dot{w} + m_o k_1 u\dot{\theta}, \\ M &= M_1 - m_o(k_1 - k_2)\, uw - k' I_y \ddot{\theta}, \end{aligned} \tag{3}$$

and,

$$\begin{aligned} Z_1 &= Z_1(w, \dot{\theta}, \delta) = Z_\delta \delta + M_2(w, \dot{\theta}), \\ M_1 &= M_1(w, \dot{\theta}, \delta) = \cdot M_\delta \delta + M_2(w, \dot{\theta}). \end{aligned} \tag{4}$$

Z_2 and M_2 are nonlinear functions of w and $\dot{\theta}$, and Z_δ and M_δ are stern plane coefficients obtained by an appropriate linearization.

HYDRODYNAMIC CHARACTERISTICS

With the assumptions of the preceding section, it is seen that the important hydrodynamic characteristics which must be determined are $Z_2(w, \dot{\theta})$ and $M_2(w, \dot{\theta})$, as well as the stern plane coefficients Z_δ and M_δ. The latter depend on how the equations are linearized with respect to δ.

The lift $Z(\delta)$, and moment $M(\delta)$, associated with a given stern plane angle δ may be determined for zero angle of attack, or transverse velocity w, and for zero turning rate $\dot{\theta}$. When so determined, the slopes $dZ(\delta)/d\delta$ and $dM(\delta)/d\delta$ are often found to be quite constant throughout a large range of δ (Ref. 8) and the values of these slopes at $\delta = 0$ are the stern-plane coefficients Z_δ and M_δ. If, however, the angle of attack (for example) is not zero, and similar measurements are made, it is found that the slopes Z_δ and M_δ are not essentially constant throughout the region of interest. One may, as has been done in Ref. 5, average the coefficients over a specified range of the variables w and $\dot{\theta}$. It is less arbitrary, although perhaps also less satisfactory, to determine Z_δ and M_δ by the first method and this is what has been done in the present treatment.

The characteristics $Z_2(w, \dot{\theta})$ and $M_2(w, \dot{\theta})$ are nonlinear functions of w and $\dot{\theta}$ and may be expressed graphically or as polynomials in w and $\dot{\theta}$. The general character of the functions seems well known, although there seems to be some doubt about the reproducibility of the results from one measurement to another. Furthermore, in the present instance there are incomplete experimental data available relating to the specific tail configuration of interest. A certain amount of effort has been directed towards the prediction of the linear terms of these functions, e.g., Refs. 6 and 7. It is pointed out in Ref. 6 that a crude but fairly reliable simplification of the problem results from the assumption that the lift Z_1 can be split into two components, as has been done in Eq. 4, one due to the deflection of the stern plane and one due to the tail fins which act as lifting surfaces. The latter component will be proportional to the local angle of attack at the tail and can be estimated using a result of aerodynamic lifting theory. The first component will be proportional to the stern plane angle and must be determined empirically. Thus one may write (see, for example, Ref. 6),

$$Z_1 = \delta Z_\delta + \alpha_{tail} Z_{tail} \tag{5}$$

where,

$$Z_{tail} = Z_t = -\frac{2\pi a_t A_t}{a_t + 2} \cdot \frac{1}{2}\rho \ell^2 V^2 , \tag{6}$$

and,

a_t - Effective-aspect-ratio of tail,

A_t = Dimensionless area of tail.

Furthermore, under the simplifying assumption that the dimensionless distance from the c.g. to the tail is one-half, then the angle of attack at the tail is just,

$$\alpha_{tail} = w/V + \dot{\theta}\ell/2V. \tag{7}$$

Thus,

$$Z_1 = \delta Z_\delta + wZ_t/V + \dot{\theta}\ell Z_t/2V . \tag{8}$$

Similarly, the moment M_2 is taken to be,

$$M_1 = \delta M_\delta + (wZ_t/2V + \dot{\theta}\ell Z_t/4V)\,\ell \tag{9}$$

since the factor $\ell/2$ represents the moment arm.

Except then for the two empirically determined coefficients Z_δ and M_δ, one could in principle calculate all terms occurring in the two linearized differential equations from a knowledge of the geometry of the body alone. In practice, however, these results are not sufficiently reliable and use is made of information obtained from hydrodynamic dynamometer tests. In order to relate this information to the above estimated coefficients, it is necessary to note that what is measured in the captive model tests includes all hydrodynamic terms but does not include the acceleration terms relating to the body alone.

When Eqs. (8) and (9) are introduced into Eqs. (3) and (4), the following results are obtained:

$$\begin{aligned} Z &= -m_o k_2 \dot{w} + \delta Z_\delta + wZ_t/V + \dot{\theta}(m_o k_1 u + \ell Z_t/2V), \\ M &= -k' I_y \ddot{\theta} + \delta M_\delta + w\left[Z_t \ell/2V - m_o(k_1 - k_2)\,u\right] + \dot{\theta}\ell^2 Z_t/4V . \end{aligned} \tag{10}$$

Therefore,

$$\begin{aligned} Z_w &= Z_t/V, \\ M_w &= m_o u(k_2 - k_1) + \ell Z_t/2V , \\ Z_{\dot{\theta}} &= m_o k_1 u + \ell Z_t/2V , \\ M_{\dot{\theta}} &= \ell^2 Z_t/4V . \end{aligned} \tag{11}$$

In dimensionless notation and taking $k_1 \cong 0$, $k_2 \cong k' \cong 1$ these results take the form

$$Z'_w = -\frac{2\pi a_t A_t}{a_t + 2},$$

$$M'_w = \frac{1}{2} Z'_w + m'_o, \qquad (12)$$

$$Z'_q = \frac{1}{2} Z'_w,$$

$$M'_q = \frac{1}{4} Z'_w.$$

The foregoing considerations should be viewed as suggestive rather than determinative and if the free-running tests appear to be more consistent with slightly different linearized coefficients, such coefficients would have just as much logical justification as the "predicted" coefficients of Eq. (12).

As a starting point, however, it is convenient to calculate coefficients using Eq. (12) to modify these in a consistent way on the basis of captive model tests, and then perhaps to modify them again on the basis of free-running tests.

The application of these considerations may now be made to the tests of the free-running model of the Hydrodynamics Laboratory. The model is scaled down 1:100 from the SST, Scheme IV submarine. No hydrodynamic tests have been made on the model and all hydrodynamic tests which have been made using captive models of the prototype are based on tail assemblies that differ to some extent from the tail of the Hydrodynamics Laboratory model.

The alternate tail "B" (for which test results are available) and the model tail may be considered to be roughly similar in size and shape and one might expect that, although the model tail has a somewhat larger projected area, the experimentally determined coefficients for the alternate tail "B" could be used to estimate the coefficients for the model tail. The question arises as to what sort of correction factor should be applied. Insofar as it does not seem to be possible to precisely calculate coefficients

from a knowledge of the geometry, the method employed results in an estimate only.

The result of applying the simplified theory to the alternate tail "B", using a projected area of 299.4 ft^2 and an aspect ratio of 2.28, is shown in Table 1, which also contains the tabulated experimental results on tail "B". The approximate ratio of the calculated results to the experimental results for tail "B" is taken as a correction factor by means of which "predicted" coefficients for the model tail are obtained from calculated coefficients. A third set of coefficients labeled "revised" is shown and will be explained later. Stern plane coefficients are also shown in Table 1. For tail "B" these are the measured coefficients. For the model tail these are calculated from the steady-state turning results by the method outlined in a later section of the paper.

TABLE 1.

Coefficients for the SST, Scheme IV.

	Alternate Tail "B"		Correction Factor	Hydrodynamics Lab. Model Tail		
	Calculated	Measured*		Calculated	Predicted	Revised
Z'_w	-.0249	-.0250	1.00	-.0277	-.0277	-.02832
M'_w	+.00596	+.0075	0.80	+.0044	+.0055	+.00625
Z'_q	-.01245	-.0083	1.50	-.01385	-.00923	-.00863
M'_q	-.00622	-.0065	1.00	-.00693	-.00693	-.00625
Z'_δ	-	-.0070	-	-	-.0090	-.00697
M'_δ	-	-.0040	-	-	-.00514	-.00412

*The measured coefficients on tail "B" were obtained by estimating the slopes (at the origin) of the hydrodynamic characteristics given in Ref. 7. The values of Z_δ and M_δ are contained in Ref. 8.

The preceding discussion related to the linear terms of the nonlinear functions $Z_2(w, \dot{\theta})$ and $M_2(w, \dot{\theta})$. The nonlinear terms have in some cases been determined experimentally and application of these terms to the differential equations has been made, e.g., Ref. 5. However, for the model

configuration these nonlinearities can only be inferred from a knowledge of the nonlinearities for a closely similar configuration. In a later section of this paper a method of constructing these nonlinearities from the trajectories of the free-running model is indicated.

The stability coefficients (at infinite speed), σ_1 and σ_2, have been calculated using the "revised" coefficients. They have the values $\sigma_1 = -0.465$, $\sigma_2 = -3.835$.

ANALYSIS

Solution of the Linearized Equations

The preceding section has shown how all terms may be estimated for the linearized equations. These were not, however, put in dimensionless form. When written in the customary dimensionless form they are:

$$\begin{aligned} m_2' \, d\alpha/ds - \alpha Z_\alpha' - q'(m_o' + Z_q') &= Z_\delta' \, \delta(s) , \\ n_y' \, dq'/ds - \alpha M_\alpha' - q' M_q' &= M_\delta' \, \delta(s) , \end{aligned} \tag{13}$$

where α and δ are measured from the values of α and δ corresponding to a steady, horizontal, neutrally buoyant run. All the coefficients Z_α', etc., are measured for this value of α and for $q' = 0$. For convenience, the equations have been rewritten in the form,

$$\begin{aligned} dx_1/ds + a_{11}x_1 + a_{12}x_2 &= c_1(s), \\ dx_2/ds + a_{21}x_1 + a_{22}x_2 &= c_2(s), \end{aligned} \tag{14}$$

where $x_1(s) = \alpha(s)$, and $x_2(s) = q'(s)$.

These have been solved and, also, $\theta(s) = \int q'(s)ds$ has been computed for $\delta(s)$ of the form,

$$\delta(s) = \begin{cases} \delta_o s/\tau , & 0 \leq s \leq \tau , \\ \delta_o , & s > \tau . \end{cases} \tag{15}$$

The solutions involve the stability exponents (at infinite speed) σ_1 and σ_2, as well as various other constants derived from the a's and c's. The solution is straightforward and only the results corresponding to the initial

conditions, $x_1(0) = x_2(0) = 0$ will be given here.

$$x_{1_2}(s) = \begin{cases} (\delta_o/\tau\left[C_{1_2}(e^{\sigma_1 s} - \sigma_1 s - 1) + D_{1_2}(e^{\sigma_2 s} - \sigma_2 s - 1)\right], & 0 \leq s \leq \tau, \\ x_{1_2}(\tau) + (\delta_o/\tau)\left[C_{1_2}(e^{\sigma_1\tau} - 1)(e^{\sigma_1(s-\tau)} - 1) + D_{1_2}(e^{\sigma_2\tau} - 1)\right. \\ \left.(e^{\sigma_2(s-\tau)} - 1)\right], & s > \tau, \end{cases} \tag{16}$$

$$\theta(s) = \begin{cases} (\delta_o/\tau)\left[(C_2/\sigma_1)(e^{\sigma_1 s} - \sigma_1 s - 1) + (D_2/\sigma_2)(e^{\sigma_2 s} - \sigma_2 s - 1)\right. \\ \left. + \frac{1}{2} B_2 s^2\right], & 0 \leq s \leq \tau, \\ \theta_2(\tau) + B_2\delta_o(s-\tau) + (\delta_o/\tau)\left[(C_2/\sigma_1)(e^{\sigma_1\tau} - 1)(e^{\sigma_1(s-\tau)} - 1)\right. \\ \left. + (D_2/\sigma_2)(e^{\sigma_2\tau} - 1)(e^{\sigma_2(s-\tau)} - 1)\right], & s > \tau. \end{cases} \tag{17}$$

The stability exponents are,

$$\sigma_{1,2} = -\frac{1}{2}(a_{11} + a_{22}) \pm \sqrt{(a_{11} + a_{22})^2/4 - (a_{11}a_{22} - a_{12}a_{21})}. \tag{18}$$

The constants which enter into the solution are

$$C_{1_2} = \frac{c_{1_2} + (a_{22_{11}} c_{1_2} - a_{12_{21}} c_{2_1})/\sigma_1}{\sigma_1(\sigma_1 - \sigma_2)}$$

$$D_{1_2} = \frac{c_{1_2} + (a_{22_{11}} c_{1_2} - a_{12_{21}} c_{2_1})/\sigma_2}{\sigma_2(\sigma_2 - \sigma_1)} \tag{19}$$

$$B_{1_2} = (a_{22_{11}} c_{1_2} - a_{12_{21}} c_{2_1})/\sigma_1\sigma_2$$

If the initial conditions $\left[x_1(0) \text{ and } x_2(0)\right]$, are not zero, correction terms must be added; however, in the model tests any deviation from zero initial conditions is experimentally not detectable and all the calculations assume $x_1(0) = x_2(0) = 0$. The steady-state values of $x_1(s)$ and $x_2(s)$ for a constant stern-plane angle δ_o are easily shown to be

$$x_{1_2}(\infty) = B_{1_2}\,\delta_o\,. \tag{20}$$

The steady-state turning rate $x_2(\infty) = q'(\infty)$ is related to the steady-state turning radius $R(\infty)$ by virtue of the relation

$$q'(\infty) = 1/R(\infty)\,, \tag{21}$$

where R is measured in ship lengths.

<u>Treatment of the Nonlinear Equations</u>

If the functions Z_2 and M_2 are not linearized, then the dimensionless equations corresponding to Eqs. (13) might be written in the form

$$\begin{aligned} d\alpha/ds - \left[Z_2'(\alpha, q') + m_o' q'\right]/m_2' &= Z_\delta'\,\delta(s)/m_2' \\ dq'/ds - M_2'(\alpha, q')/n_y' &= M_\delta'\,\delta(s)/n_y' \end{aligned} \tag{22}$$

In the nonlinear form of these equations, the functions $\left[Z_2'(\alpha, q') + m_o' q'\right]/m_2'$ and $M_2'(\alpha, q')$ may be represented graphically as functions of α for different values of q'.

In the vicinity of $\alpha = q' = 0$ these functions will be straight lines whose slopes and intercepts are determined by the coefficients Z_α', M_α', Z_q' and M_q'. The trajectories of the model are used to estimate the shape of the hydrodynamic characteristics when α and q' are not small. It is likely that trajectories which are predicted from characteristics constructed in this way will only be approximate and that the approximation is less likely to be satisfactory the more the predicted trajectory differs from those used in constructing the hydrodynamic characteristics.

APPLICATION TO THE MODEL

Several sets of calculations have been carried out to construct the

hydrodynamic characteristics Z_2 and M_2 in such a way that trajectories calculated from them will give results which are consistent with the experimental results.

It is characteristic of the free-flight tests that there is no very sensitive test of the validity of the linear terms of these functions. This is, perhaps, mainly due to the technical limitations of the free-flight tests, i.e., trajectories which one might conceive of as being always within the linear range, and preferably steady-state trajectories, are not possible within the limited space of the testing tank. On the other hand, those trajectories which are possible, and which have reasonably small scatter, have only a very limited "linear" range which is not adequate to test the predicted or assumed linear terms.

It is also characteristic of the free-flight tests that the two dependent variables of the differential equations, namely the angle attack α and the angular turning rate q', are not measured directly but can only be derived from other measured quantities. The turning rate q' is the derivative of the body inclination θ which one can measure, and the angle of attack is even less precisely known since it is measured by the difference between the body inclination θ, and the inclination of the trajectory $\tan^{-1} dz/ds$. The scatter in the values of α and q' from presumably identical trajectories will therefore be much greater than the scatter in the values for the inclination θ and the depth z.

It might justifiably be argued that one is only interested in the inclination and the depth, and that if differential equations can be constructed which enable one to predict these quantities, then one need not be concerned about the logical basis for the equations. One must, however, always keep in mind the danger of extrapolation from such results. It has been pointed out that the "predicted" coefficients given in Table 1 for the model are not predicated by any very compelling reasons. On the other hand, the data obtained from the trajectories are insufficient to allow one to derive these coefficients. As a consequence, the selection of an appropriate set of linear terms for the characteristics is reduced to a trial and error procedure. Calculations (of a type to be described) were made using the "predicted" coefficients and using several sets of revised coefficients. The "revised" coefficients shown in Table 1 are therefore somewhat arbitrary.

However, they lead to results which, within a limited range, appear to be quite reasonable. Only the results of calculations using the "revised" coefficients will be presented here.

Figure 1 shows the steady-state turning rate, $q'(\infty)$ and angle of attack, $\alpha(\infty)$ for a constant stern plane angle $\delta_s = \delta_o$. The dashed curve is the experimental curve obtained with the model for dives. The straight lines correspond to Eq. (20). Numerically,

$$q'(\infty) = -1.42\ \delta_o, \quad \text{and}$$

$$\alpha(\infty) = -0.820\ \delta_o .$$

The first value is obtained by approximating the slope of the experimental curve. The latter value is obtained by taking, somewhat arbitrarily, the value 0.577 for the ratio Z_δ'/M_δ'. The solid curves of Fig. 1 are smooth curves constructed to approach the straight lines in the linear region. In the case of the curve for $q'(\infty)$ the curve is constructed to approximate the experimental curve. In the case of the curve for $\alpha(\infty)$ the experimental data are not shown because of the large scatter and the curve shown is somewhat arbitrary because of this scatter.

The procedure whereby the characteristics $\left[Z_2(\alpha, q') + m_o' q'\right]/m_2'$ and $M_2(\alpha, q')/n_y'$ shown in Figs. 2 and 3 are constructed will now be outlined.

Using the "revised" coefficients given in Table 1, together with the values $m_o' = 0.01840$, $m_2' = 0.0354$ and $n_y' = 0.001787$, the linear portion of the characteristics becomes,

$$(Z_2' + m_o' q')/m_2' = -0.800\,\alpha + 0.290\,q' , \quad \text{and}$$

$$M_2'/n_y' = +3.50\,\alpha - 3.50\,q' .$$

Furthermore, $Z_\delta'/m_2' = -0.1969$ and $M_\delta'/n_y' = -2.307$. Thus, Eqs. (22) become

$$\begin{aligned} d\alpha/ds + 0.800\alpha - 0.290\,q' + \text{nonlinear terms} &= -0.1969\,\delta(s), \\ dq'/ds - 3.50\,\alpha + 3.50\,q' + \text{nonlinear terms} &= -2.307\,\delta(s). \end{aligned} \qquad (23)$$

For steady-state turning $dq'/ds = d\alpha/ds = 0$, and for each value of $\delta(s) = \delta_o$ there are values of $q'(\infty)$ and $\alpha(\infty)$ taken from Fig. 1 which, through Eqs. (23),

determine the steady-state curves in Figs. 2 and 3. The appropriate values of q' are indicated on the steady-state curve.

The dive maneuver resulting from a stern plane deflection of 18°, achieved at a 5° per second rate, has been used as a second set of data from which Z_2 and M_2 may be constructed. Figure 4, which shows the inclination $\theta(s)$, and Fig. 5, which shows the depth $z_o(s)$ include experimental and calculated curves for incline runs as well as the dive. However, only one of these, e.g., SST_M Run S-101 has been used in constructing the characteristics. Figure 4 also shows the stern plane action for the dive and incline trajectories. The short vertical lines which intersect the trajectories in Figs. 2 through 7 may be correlated with the stern plane action.

The desired information which is to be obtained from these curves consists of the angular velocity $q'(s) = d\theta/ds$, the angular acceleration dq'/ds, the angle of attack $\alpha(s)$ and the rate of change of α, $d\alpha/ds$. This information, which is shown in Figs. 6 and 7, is most satisfactorily obtained by graphical differentiation with subsequent numerical integration as a check. Other methods, including difference tables and the fitting of high degree polynomials to $\theta(s)$ and $z_o(s)$ with subsequent differentiation have been tried but do not seem to the writer to be any more satisfactory.

It is found in practice, as explained elsewhere, that the initial portion of the curves cannot be obtained with satisfactory accuracy by this procedure. Consequently, Eqs. (16) and (17) with the appropriate numerical constants obtained from the "revised" column of Table 1, have been used to compute the initial portions of the "18° dive trajectories" in Figs. 2 and 3. The trajectories are continued using the values of $d\alpha/ds$, dq'/ds, α, and q' from Figs. 6 and 7 in Eqs. (22). As before, the values of q' are marked on the trajectory.

A first approximation to the hydrodynamic characteristics may be obtained by drawing smooth curves through corresponding values of q' on the steady-state curve and the 18° dive curve. Logically, it would seem to be more reasonable to repeat this procedure for the other dives (15°, 12°, 9°, etc.) and in this way to obtain several points on the characteristic curve corresponding to each value of q'. However, the data are not quite accurate enough to make this procedure as satisfactory as it seems to be to construct the smooth curves by judgment based on similar experimentally

obtained hydrodynamic characteristics.

Having constructed the hydrodynamic characteristics, it is then possible to compare a new computed trajectory with one obtained experimentally. For this comparison, the incline trajectory obtained by throwing the sternplane to 18° at 5°/sec, holding for 4.8 seconds (prototype), and returning to neutral at 5° sec, was used. The computed trajectory is shown, together with two of the experimental trajectories SST_M Runs 196 and 197, in Figs. 4 and 5. The computed trajectory was obtained by a step-by-step numerical integration using values of Z_2 and M_2 obtained from Figs. 2 and 3. Experimentally it is difficult to run an incline trajectory which exactly repeats the initial portion of the dive trajectory as it theoretically should do; consequently, one cannot expect to obtain much better agreement than that shown.

The logical extension of this procedure would be to calculate a "pullout" trajectory using the incline trajectory up to $s = 2.56$, and to continue the calculations with a reverse stern plane action for a prescribed time duration. One would, of course, have to extrapolate the hydrodynamic characteristics in some way.

This has been attempted; however the method has a practical limitation since the numerically integrated solution tends to diverge from the true solution and if this divergence becomes too large the "solution" begins to oscillate and no longer gives a true prediction. This could perhaps be avoided at some expense of effort; however, because of the arbitrariness involved in the extrapolation of the hydrodynamic characteristics, it does not seem to be justified.

SUMMARY AND CONCLUSIONS

The paper represents an attempt to make a theoretical analysis of free-running trajectories of a model experimental submarine, the SST, Scheme IV. The basic assumptions on which the theory is based are set forth in some detail. The equations of motion are not derived, this aspect of the problem having been adequately covered elsewhere. However, the various forms which the equations may take are indicated and explained, particularly with reference to the hydrodynamic forces and moments involved. These hydrodynamic terms are discussed separately at some length to form a basis for the estimation of numerical values for the model. Finally, the analysis consists essentially in the construction of hydrodynamic characteristics using available information from both static tests and free-running tests. The use of these characteristics to predict new trajectories is demonstrated.

There is at present rather too much scatter in the data from the free-running tests to allow of its being systematically used in the way in which it has been used in the analysis reported here. However, with more accurate data it does seem possible that more extended results of the form reported here could be obtained and that such data might have value for design purposes.

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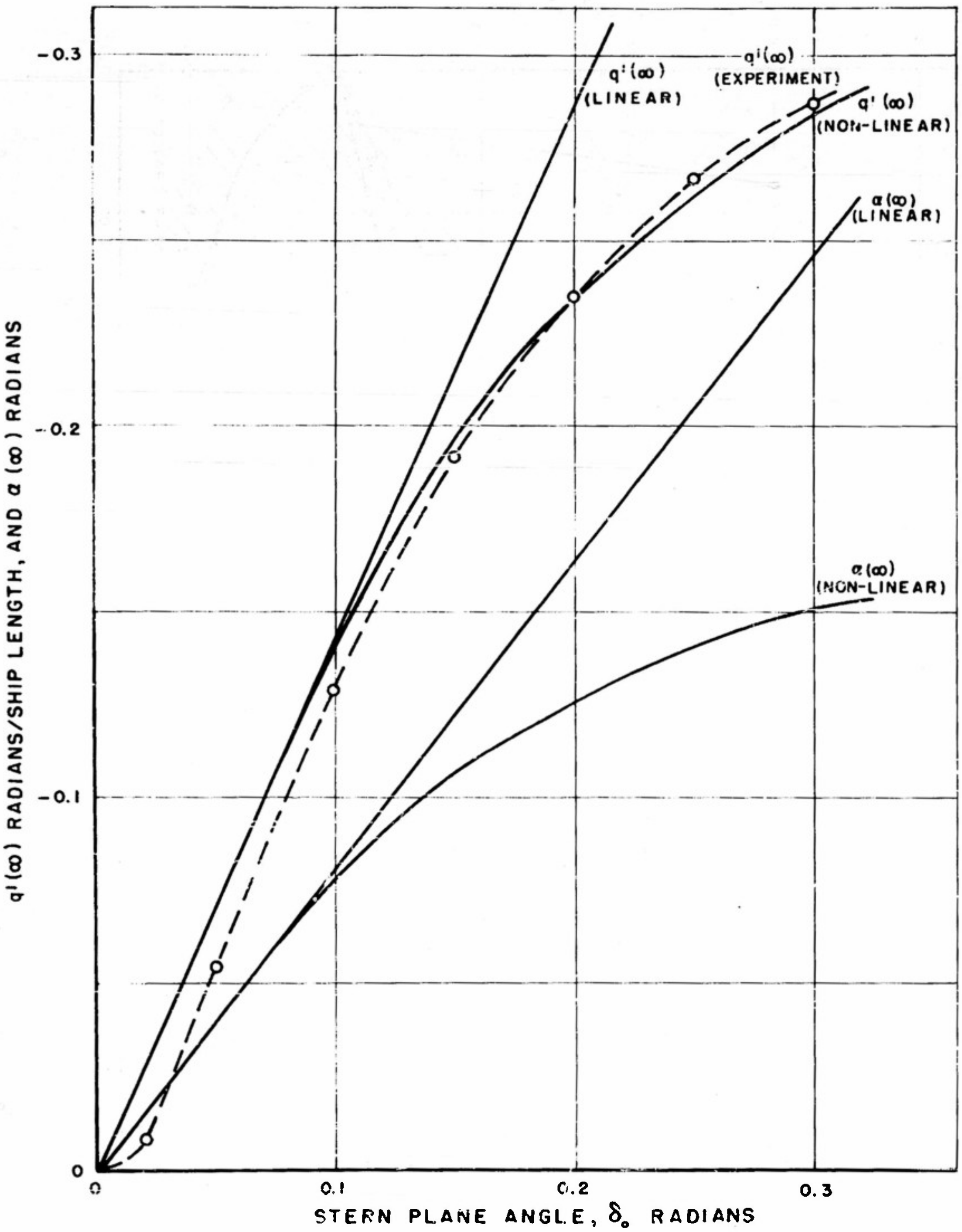

Fig. 1 - Steady-state turning rate $q'(\infty)$ and angle of attack $\alpha(\infty)$.

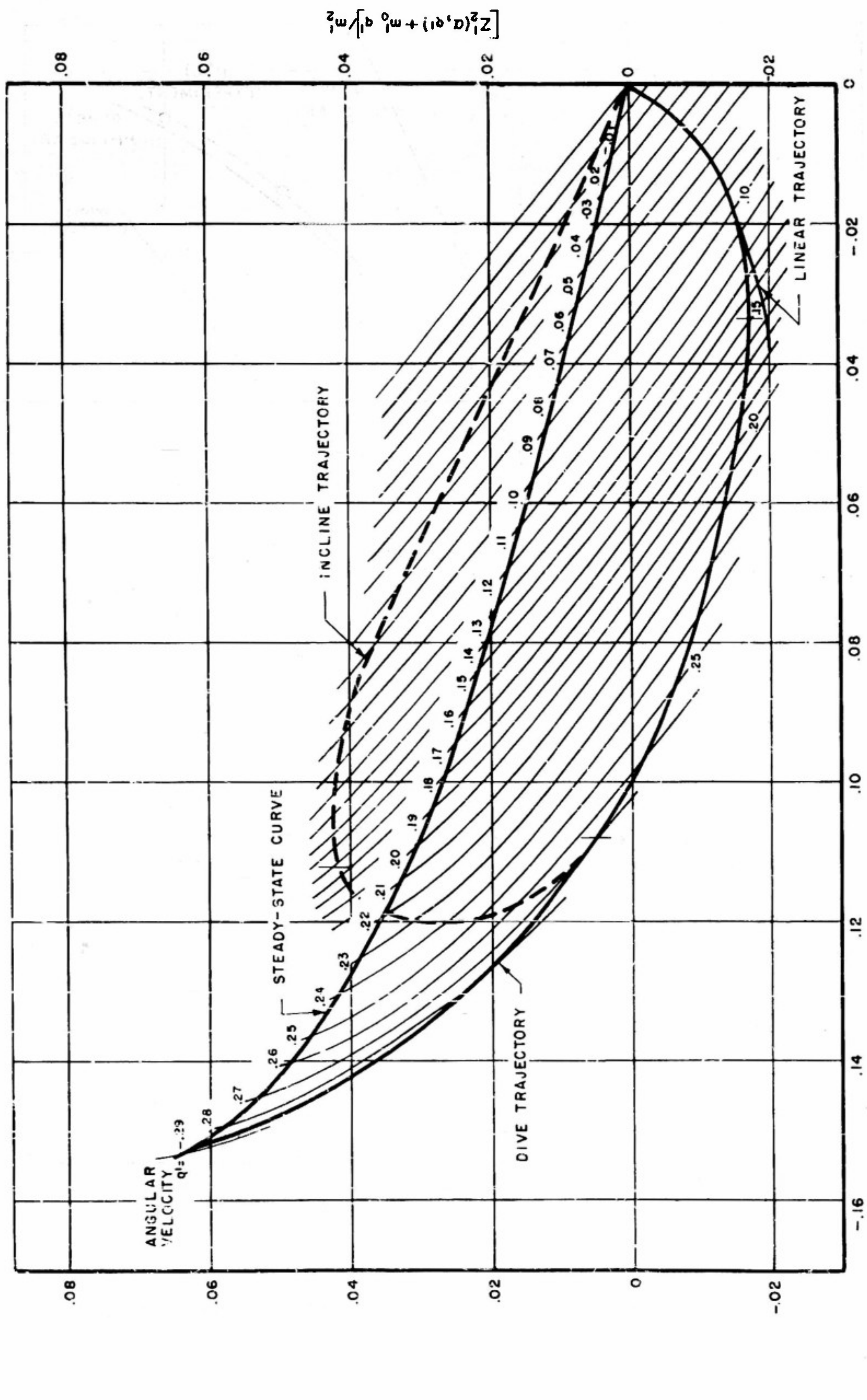

Fig. 2 - Total hydrodynamic lift vs. angle of attack for various angular velocities. (All quantities dimensionless).

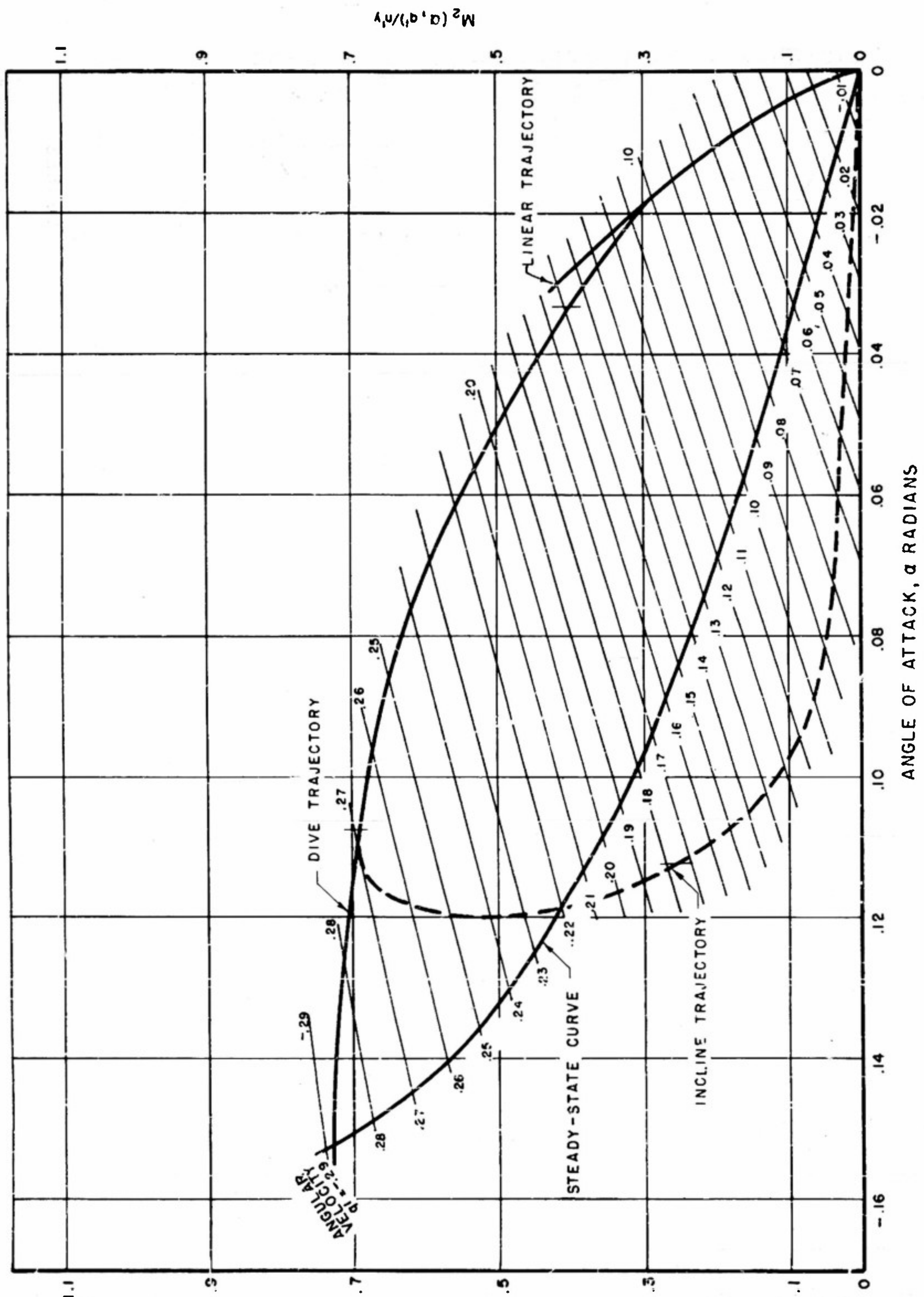

Fig. 3 - Hydrodynamic moment vs. angle of attack for various angular velocities. (All quantities dimensionless).

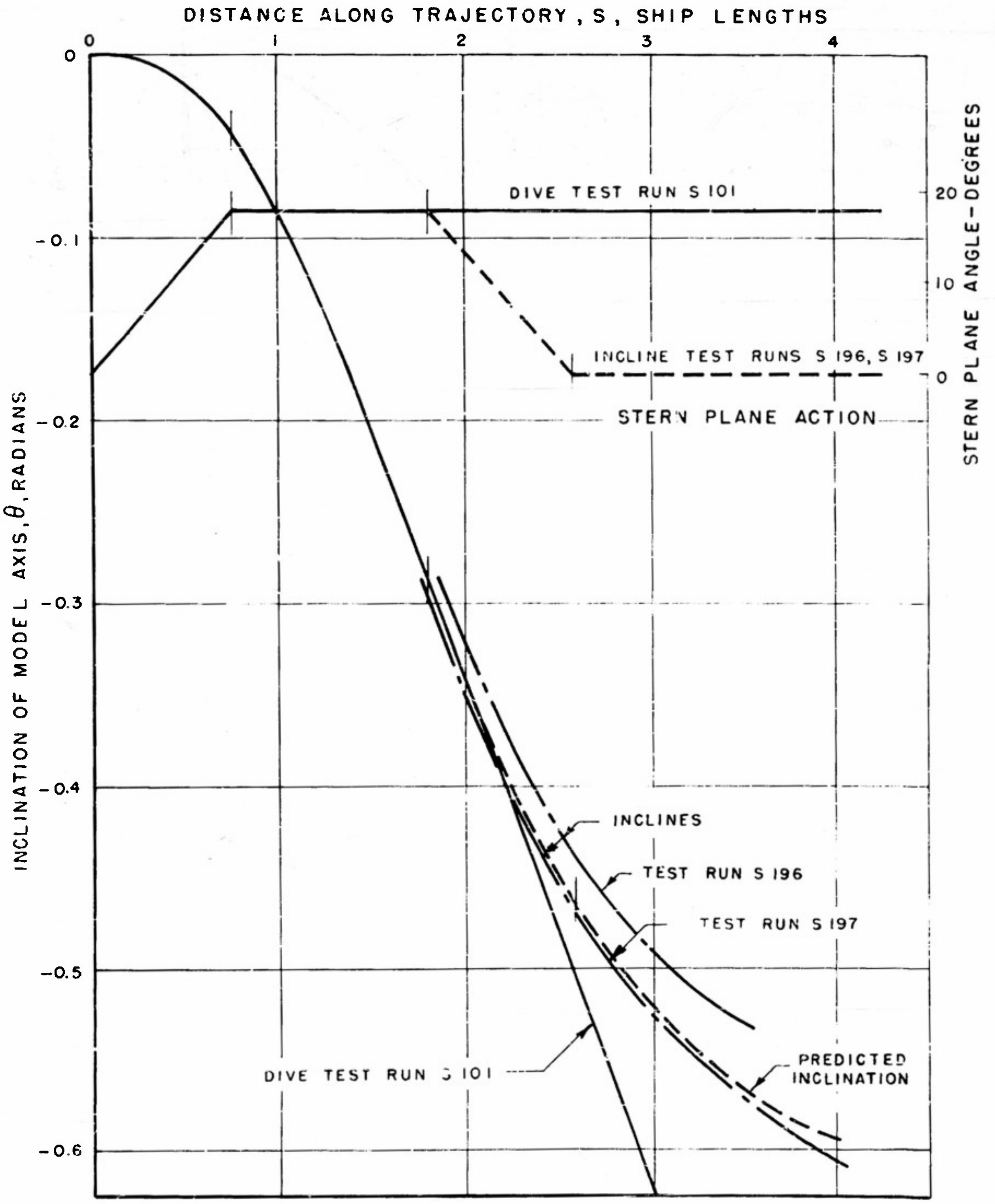

Fig. 4 - Inclination of model axis resulting from stern-plane actions shown.

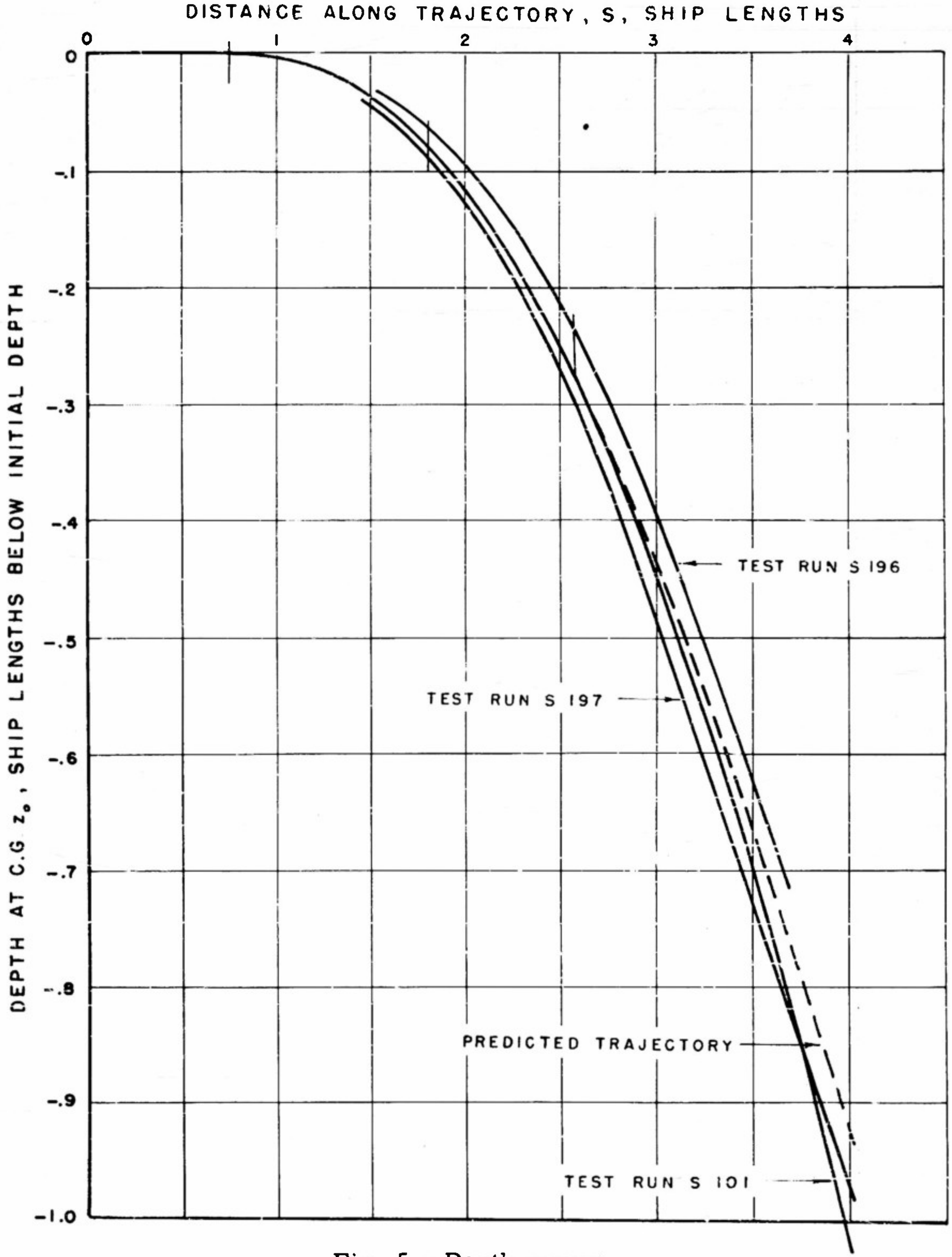

Fig. 5 - Depth curves.

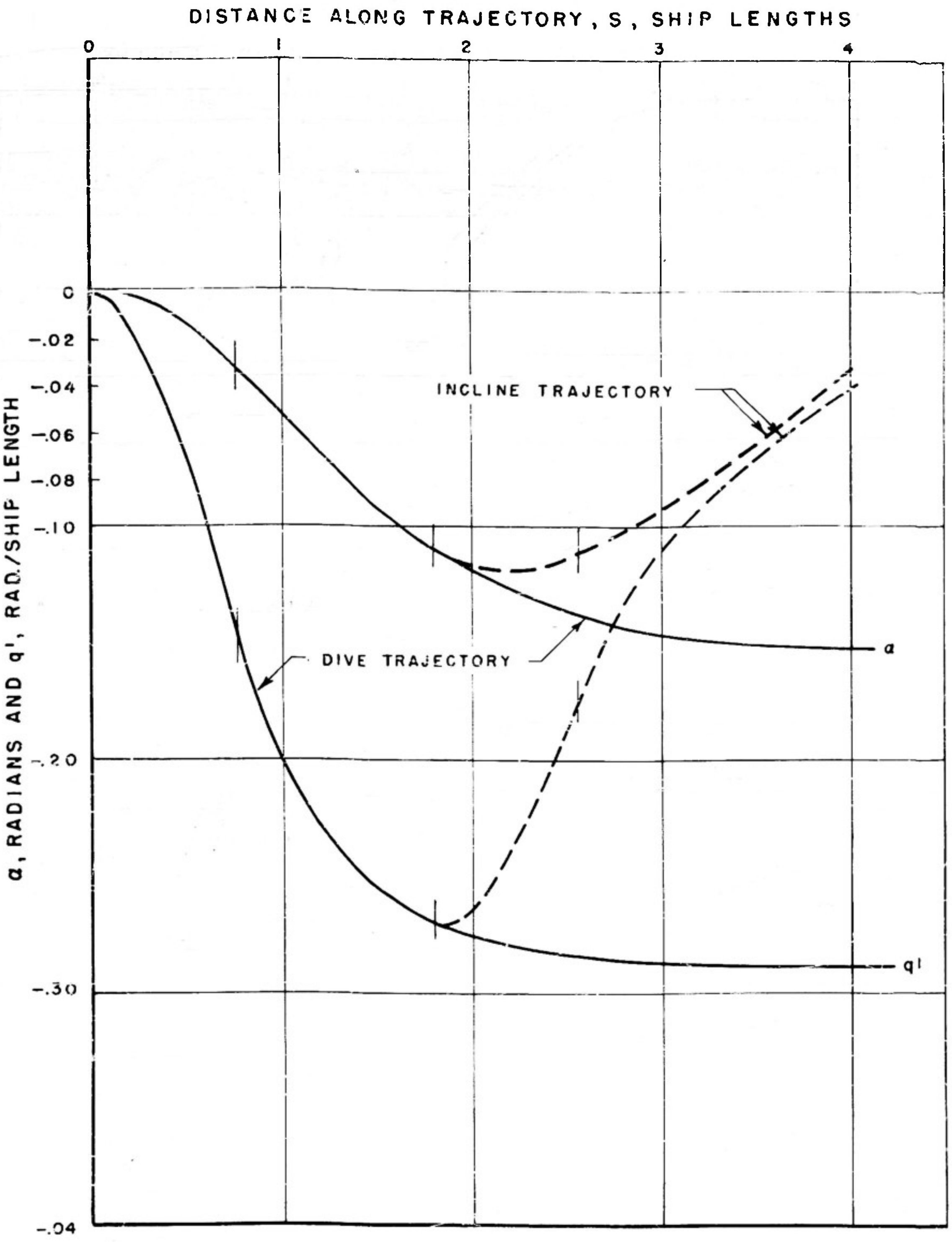

Fig. 6 - Angular velocity q' and angle of attack α.

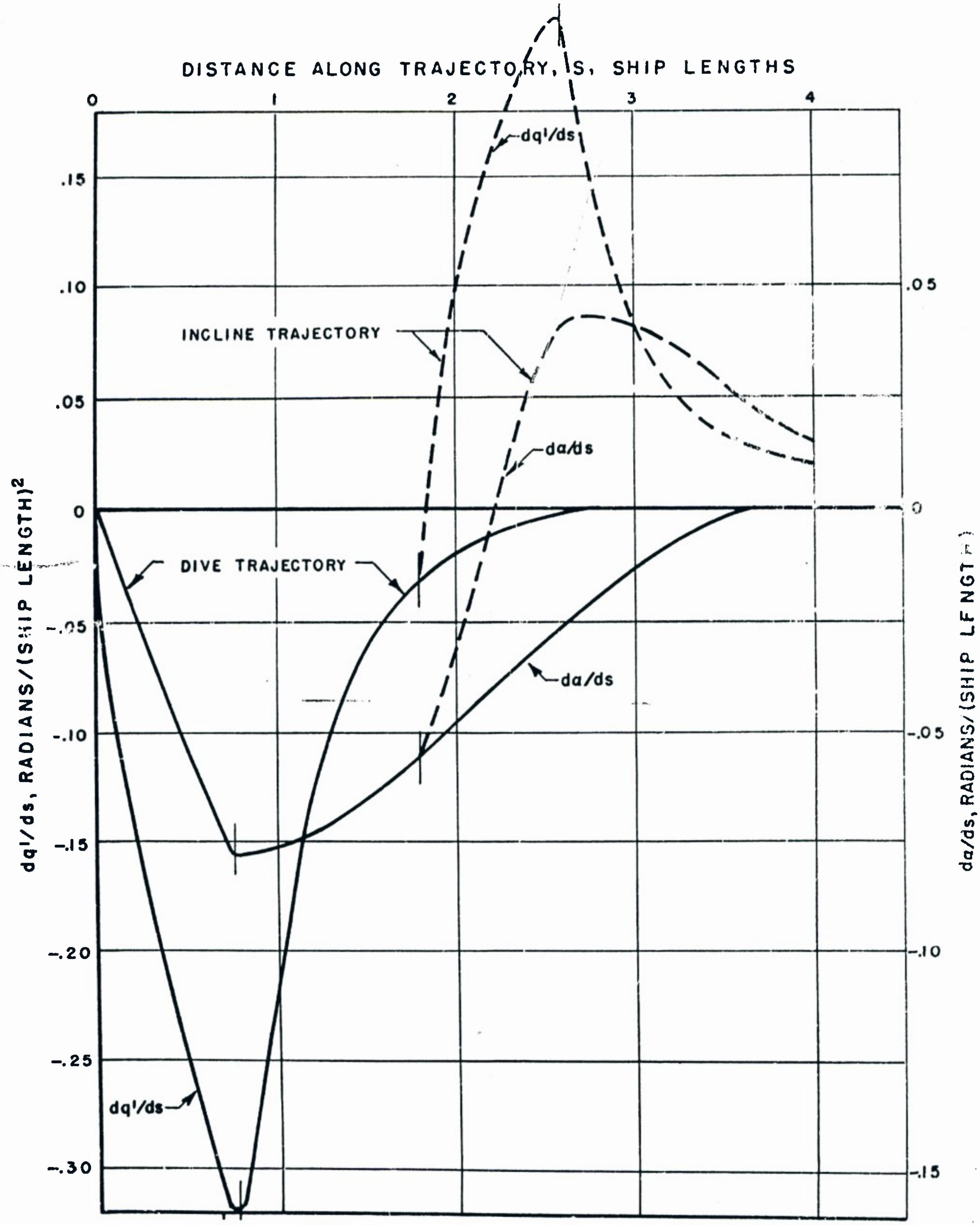

Fig. 7 - Angular acceleration dq'/ds and rate of change of angle of attack $d\alpha/ds$.

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